Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 16

Time: 3 Hours **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. Find the HCF of 336 and 54

A. 9
B. 6
C. 8
D. 12

2. Find the discriminant of the following equation: $3x^2 - 3x + 8 = 0$

A. 92
B. 96
C. −92
D. −96

3. Which of the following is the zero of the polynomial $x^2 - 5x + 6$.

A. 2

B. −1

C. 0

D. −2

4. What is the value of 10^{th} term of an A.P., if its first term is p and common difference is q?

A. p

B. 10q

C. p − 9q

D. p + 9q

5. If the system of equations 3x − 2y = 0 and kx + 5y = 0 has infinitely many solutions, then what is the value of k?

A. 5

B. $-\frac{15}{2}$

C. 3

D. $-\frac{2}{5}$

6. If $P\left(\frac{a}{3}, 4\right)$ is the midpoint of the line segment joining A(−6, 5) and B(-2, 3) then the value of a is

A. 12

B. −12

C. 4

D. −4

7. Which of the following are the coordinates of a point which divides the line segment joining the points A(−3, 6) and B(5, 2) in the ratio 1 : 3?

A. (5, −3)

B. (2, 6)

C. (−1, 5)

D. (1, 3)

8. In a ΔABC, ∠A = x°, ∠B = (3x)° and ∠C = y°. If 3y − 5x = 30, then ΔABC =

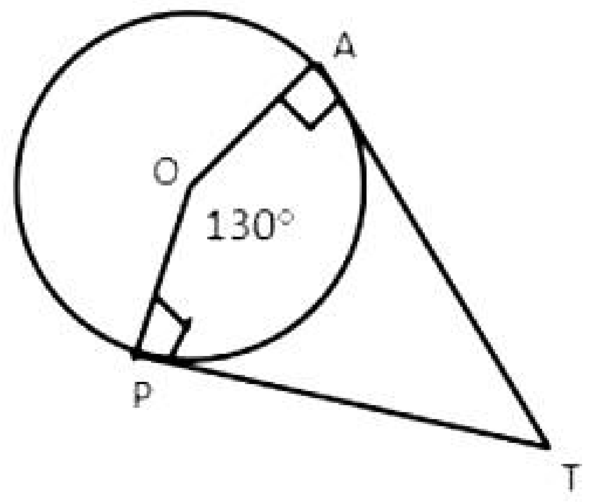
A. Right-angled

B. Acute angled

C. Obtuse angled

D. Can't determine

9. In the given figure, if TA and TP are tangents to a circle with centre O, so that $m\angle AOP = 130°$ then find $m\angle ATP$.



A. 50°
B. 130°
C. 80°
D. 260°

10. The perimeters of two similar triangles ABC and PQR are 32 cm and 24 cm, respectively. If PQ = 12 cm, then find AB.

A. 24 cm
B. 32 cm
C. 12 cm
D. 16 cm

11. For which angle, the trigonometric function of tan is not defined?

A. 30°
B. 90°
C. 60°
D. None of the above

12. Find the value of $\cos 1° \cos 2° \cos 3° \ldots \cos 180°$

A. 0
B. 1
C. −1
D. None of the above

13. Find the value of θ if $\cos\theta = \sin\theta$.

A. 30°
B. 45°
C. 60°
D. 90°

14. The length of a chain used as the boundary of a semi-circular park is 90 m. Find the area of the park.

A. $481.55\ m^2$
B. $481.35\ m^2$
C. $481.25\ m^2$
D. $481.15\ m^2$

15. The volume of a hemisphere is given by the formula

A. $\frac{2}{3}\pi r^3$

B. $\frac{2}{3}\pi r^2$

C. $\frac{2}{3}r^3$

D. $\frac{2}{3}r\pi^3$

16. Find the median of the following data:

3, 11.5, 5, 2.1, 6, 8.92, 7

A. 5

B. 5.4

C. 5.6

D. 6

17. Find the mean:

Class	Frequency
10–20	11
20–30	15
30–40	20
40–50	30
50–60	14
60–70	10

A. 30

B. 30.10

C. 40

D. 40.10

18. A cylindrical pencil sharpened at one edge is the combination of

A. A cone and a cylinder

B. Frustum of a cone and a cylinder

C. A hemisphere and a cylinder

D. Two cylinders

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19. Statement A (Assertion): 250 lottery tickets were sold and there are 5 prizes on these tickets. If Kunal has purchased one lottery ticket, then the probability that he wins a prize is $\frac{1}{50}$.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

20. Statement A (Assertion): If one zero of $3x^2 + 8x + k$ is the reciprocal of the other only when k is equal to 1.

Statement R (Reason): For a quadratic polynomial $ax^2 + bx + c$, we have, product of zeroes $= \frac{c}{a}$

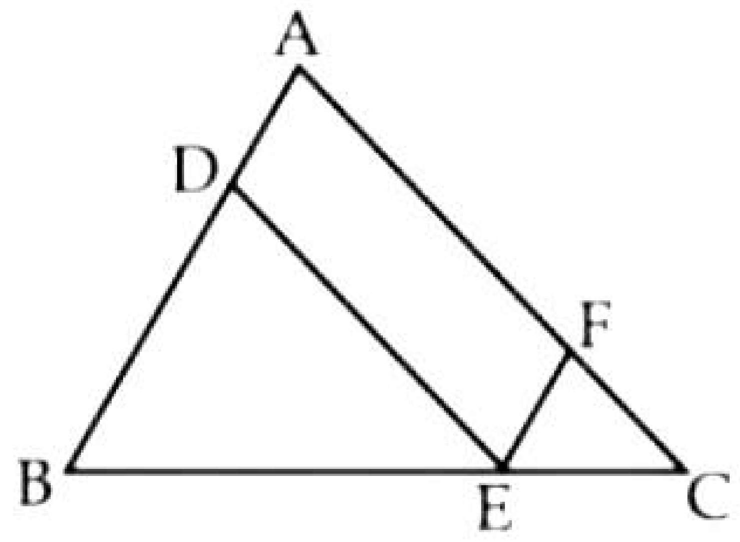
A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

Section B

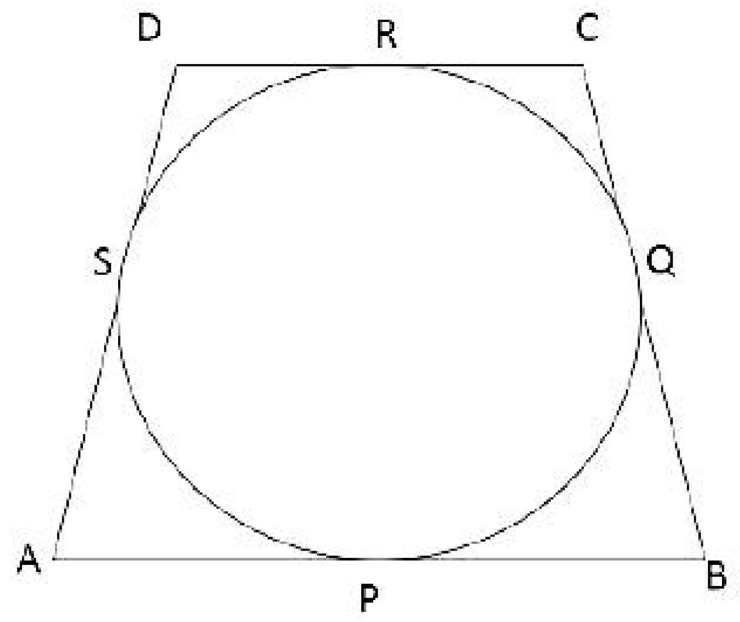
21. Mr. Shastri's cell phone PIN is dbac such that $42000 = a^4 \times b \times c^3 \times d$. Find the PIN. [2]

22. In the given figure, D, E, F are the points on sides AB, BC and AC of triangle ABC such that ADEF is a parallelogram. Prove that

$$\frac{CF}{FA} = \frac{AD}{BD}$$



23. In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD. [2]



24. Prove that: $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$ [2]

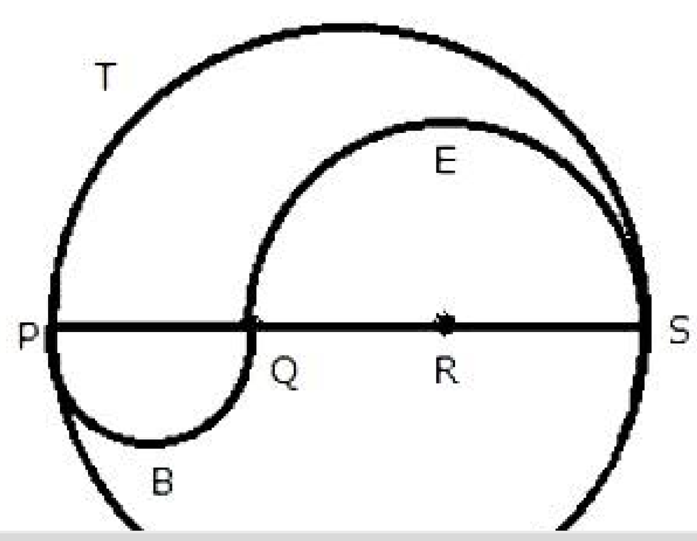
OR

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P cos P and tan P.

25. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°. [2]

OR

PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semicircles are drawn with PQ and QS as diameters, as shown in the given figure. If PS = 12 cm, find the perimeter and area of the shaded region. (Take π = 3.14)



Section C

Section C consists of 6 questions of 3 marks each.

26. The number of fruits of each kind A, B and C are 50, 90 and 110 respectively. In each basket, the equal number of fruits of same kind are to be kept. Find the minimum number of baskets required to accommodate all fruits. [3]

27. Which term of the A.P. 3, 8, 13, 18, … is 78? [3]

28. Check whether −150 is a term of the A.P. 11, 8, 5, 2, … [3]

OR

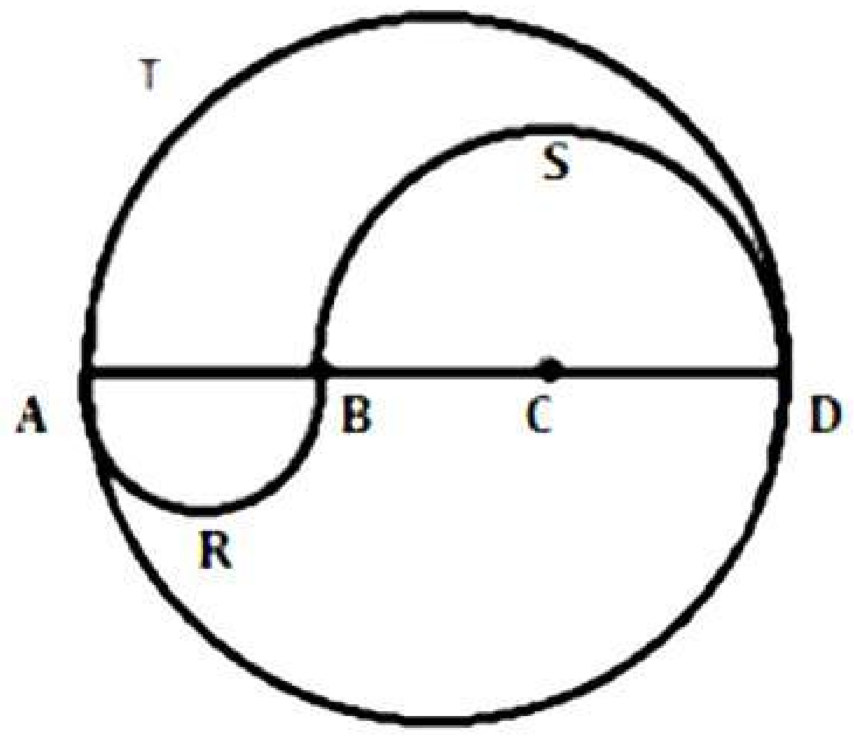
Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and the 16^{th} term is 73.

29. If $x = \cot A + \cos A$ and $y = \cot A - \cos A$, then show that $\left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1$ [3]

30. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 10 minutes. [3]

OR

ABCD is a diameter of a circle of radius 12 cm. The lengths AB, BC and CD are equal. Semicircles are drawn with AB and BD as diameters, as shown in the given figure. If AD = 24 cm, find the perimeter and the area of the shaded region. (Take $\pi = 3.14$)



31. After removing black kings, queens and jacks from a deck of 52 playing cards, a deck is well shuffled and a card is drawn from the remaining cards. Find the probability of getting (i) a jack, (ii) a black card, (iii) a heart card. [3]

Section D

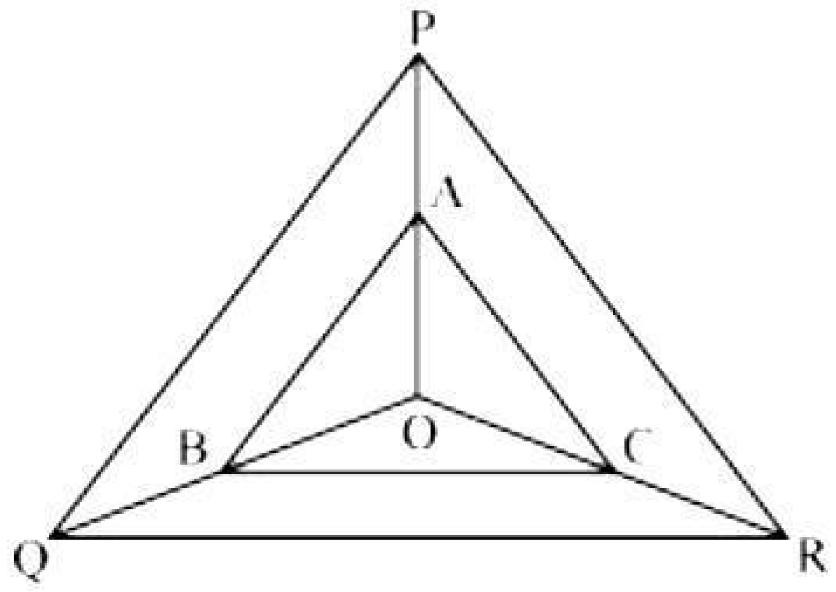
Section D consists of 4 questions of 5 marks each.

32. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age. [5]

OR

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

33. In figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR. [5]



34. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$. [5]

OR

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs. 500 per m^2.

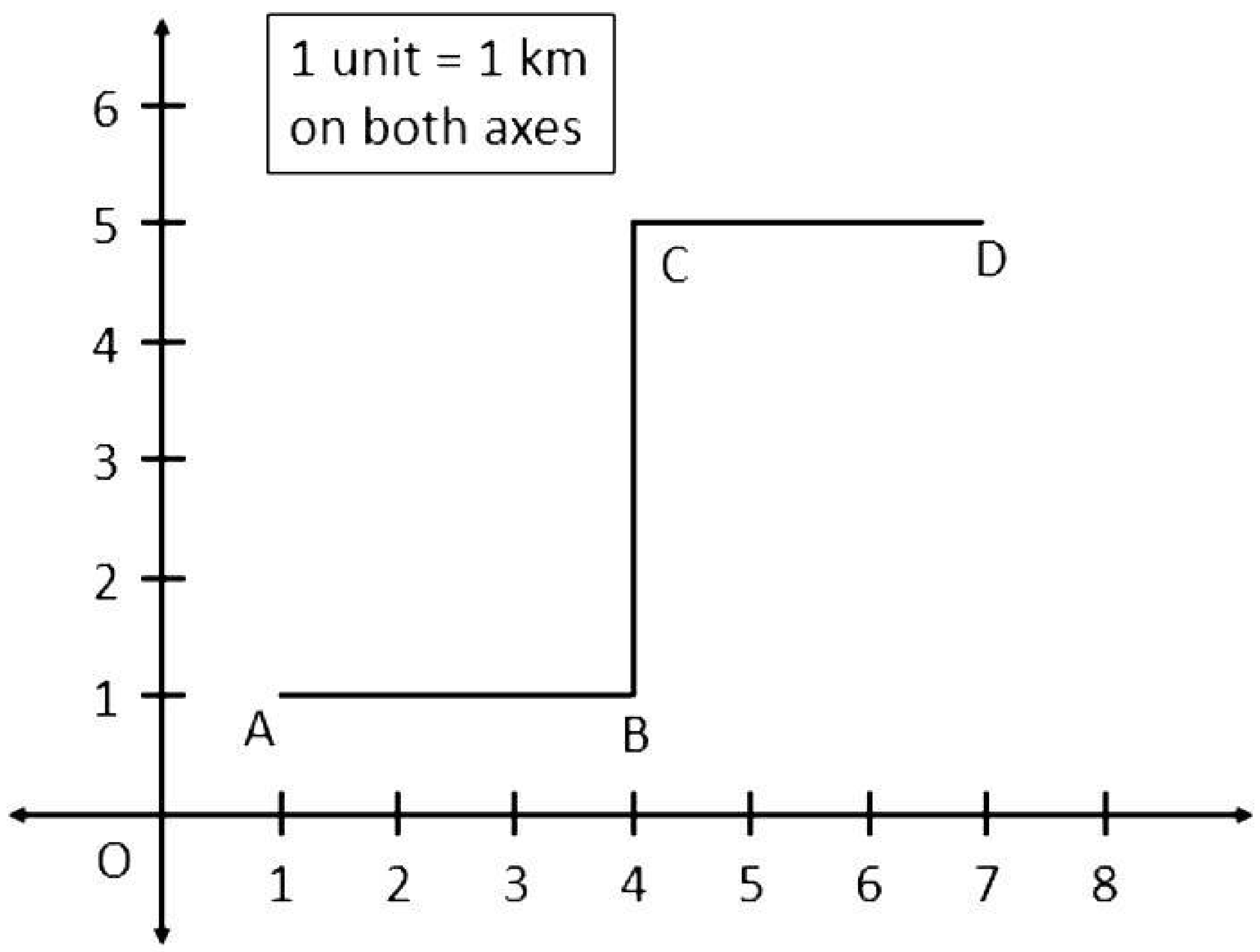
35. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent. [5]

Number of days	0 – 6	6 – 10	10 – 14	14 – 20	20 – 28	28 – 38	38 – 40
Number of students	11	10	7	4	4	3	1

Section E

Case study based questions are compulsory.

36. Amey's school has organized an inter-school race competition. The race track is constructed as shown in the diagram. Initially, the track follows a straight path AB, then it takes a 90° left and follows a straight path BC for some distance. Finally, it takes a 90° right and then follows a straight path CD to the endpoint. Now using the given information answer the following questions.



i. Find the distance AB. [1]

ii. Find the distance BC. [1]

iii. Find the distance AC. [2]

OR

Find the distance BD. [2]

37. Ajay has decided to visit different parts of his state, and to travel from one place to another, he will book a taxi each time. Now the taxi rates are fixed and it costs Rs. 15 for the first km and Rs. 8 for each additional km. Now using the given data answer the following questions.

i. Find the taxi fare for Ajay, if he travels for 4 km. [1]

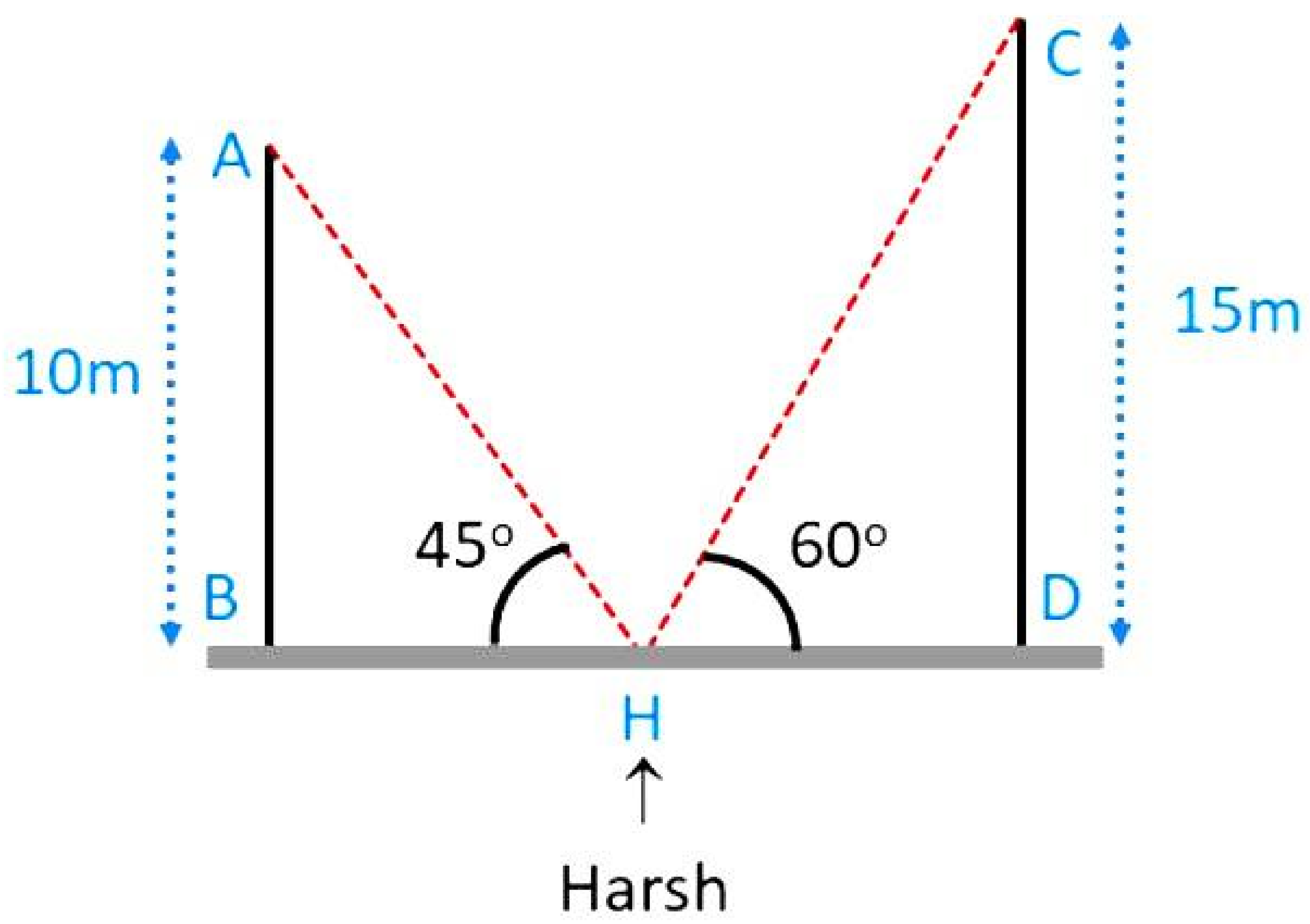
ii. If the taxi fare comes out to be Rs. 63, then find the total kilometers travelled. [2]

OR

If it takes Rs. 103 to travel from place A to B, then find the distance between places A and B. [2]

iii. If Ajay has decided to visit a place that is 5 km away, rather than visiting a place which is 3 km away, then how much would it cost extra for him? [1]

38. Harsh is standing between two buildings having heights as 10 m (AB) and 15 m (CD). Now the angle of elevation from the point (H) where Harsh is standing, to the top of building AB is 45° whereas the angle of elevation from the same point to the top of building CD is 60°. Using the given data, answer the following questions.



i. Find the distance between Harsh and building AB. [1]

ii. Find the distance between Harsh and building CD. [1]

iii. Find the length of a rope joining the points A and H. [2]

OR

Find the length of a rope joining the points C and H. [2]

Solution

Section A

1. Correct option: B

Explanation:

336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

HCF = $2 \times 3 = 6$

2. Correct option: C

Explanation:

The given equation is $3x^2 - 2x + 8 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 3, b = -2, c = 8$

$\therefore D = b^2 - 4ac = (-2)^2 - (4 \times 3 \times 8) = 4 - 96 = -92$

3. Correct option: A

Explanation:

If p(a) = 0, then it is said that 'a' is a zero of p(x).

$p(2) = (2^2) - 5(2) + 6 = 4 - 10 + 6 = 0$

So, 2 is a zero of the given polynomial.

4. Correct option: D

Explanation:

Given: a = p, d = q

10^{th} term $= a_{10} = a + 9d = p + 9q$

5. Correct option: B

Explanation:

The system of equations $3x - 2y = 0$ and $kx + 5y = 0$ has infinitely many solutions.

Here, $a_1 = 3, b_1 = -2, a_2 = k, b_2 = 5$

Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{3}{k} = \frac{-2}{5} \Rightarrow k = \frac{-15}{2}$

6. Correct option: B

Explanation:

$P\left(\frac{a}{3}, 4\right)$ is the midpoint of the line segment joining A(−6, 5) and B(−2, 3).

$\Rightarrow$ Co-ordinates of the midpoint $= \left(\frac{-6-2}{2}, \frac{5+3}{2}\right) = (-4, 4)$

7. Correct option: C

Explanation:

Let P(x, y) be the point which divides the line segment AB in the ratio 1: 3.

Therefore, $x = \frac{1(5)+3(-3)}{1+3} = -1$ and $y = \frac{1(2)+3(6)}{1+3} = 5$

Hence, the coordinates of P are (−1, 5).

8. Correct option: A

Explanation:

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow x + 3x + y = 180$

$4x + y = 180$ ....(1)

Also, $3y - 5x = 30$

$\Rightarrow -5x + 3y = 30$ ....(2)

Multiplying (1) by 3, we get

$12x + 3y = 540$ ....(3)

Subtracting (2) from (3), we get

$17x = 510 \Rightarrow x = 30$

Putting x = 30 in (1), we get

$4 \times 30 + y = 180 \Rightarrow y = 60$

Hence, $\angle A = 30^\circ$, $\angle B = 3 \times 30^\circ = 90^\circ$, $\angle C = 60^\circ$

So, the triangle is right angled.

9. Correct option: A

Explanation:

OA is perpendicular to TA by tangent radius theorem.

OP is perpendicular to TP by tangent radius theorem.

Now, $\angle ATP + \angle OAT + \angle OPT + \angle POA = 360^\circ$

$\Rightarrow 90^\circ + 90^\circ + 130^\circ + \angle ATP = 360^\circ$

$\Rightarrow m\angle ATP = 50^\circ$

10. Correct option: D

Explanation:

It is given that ΔABC and ΔPQR are similar triangles, so the corresponding sides of both triangles are proportional.

So, $\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AB}{PQ}$

Then, $\frac{AB}{12} = \frac{32}{24}$

$\Rightarrow AB = \frac{32 \times 12}{24} = 16$ cm

11. Correct option: B

12. Correct option: A

Explanation:

$$\text{L.H.S.} = \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 180^\circ$$
$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 180^\circ$$
$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \times 0 \times \ldots \cos 180^\circ \quad (\text{Since } \cos 90^\circ = 0)$$
$$= 0$$

13. Correct option: B

Explanation:

$$\cos 45^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

14. Correct option: C

Explanation:

Let the radius of the park be r metres.

Thus, $\pi r + 2r = 90^\circ \Rightarrow \frac{22r}{7} + 2r = 90^\circ$

Hence, the reason (R) is true.

$$\Rightarrow \frac{36r}{7} = 90 \Rightarrow r = \frac{90 \times 7}{36} = 17.5 \text{ m}$$

Area of semicircle $= \frac{1}{2}\pi r^2 = \left(\frac{1}{2} \times \frac{22}{7} \times 17.5 \times 17.5\right) m^2 = 481.25 \text{ m}^2$

15. Correct option: A

Explanation:

Volume of a hemisphere is given by $\frac{2}{3}\pi r^3$

16. Correct option: D

Explanation:

Arranging given data in the ascending order:

2.1, 3, 5, 6, 7, 8.92, 11.5

Number of observations = 7

So, $\left(\frac{n+1}{2}\right)^{th}$ term is the median.

$\left(\frac{7+1}{2}\right)^{th} = 4^{th}$ term is the median.

Hence, 6 is the median.

17. Correct option: D

Explanation:

We have,

Class	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
10 – 20	11	15	165
20 – 30	15	25	375
30 – 40	20	35	700
40 – 50	30	45	1350
50 – 60	14	55	770
60 – 70	10	65	650
	$\sum f_i = 100$		$\sum f_i x_i = 4010$

Mean $\bar{x} = \frac{\sum(f_i x_i)}{\sum f_i} = \frac{4010}{100} = 40.10$

18. Correct option: A

Explanation:

A pencil consists of a cylinder and a cone.

19. Correct option: B

Explanation:

Total number of tickets sold = 250

Number of prizes = 5

Let E be the event of getting a prize.

Number of favourable outcomes = 5

$\therefore$ P(getting a prize) = $P(E) = \frac{5}{250} = \frac{1}{50}$

So, the assertion is true.

As we know that the probability of an event can be zero.

So, the reason is true but it is not the correct explanation of assertion.

20. Correct option: D

Explanation:

Let α and $\frac{1}{\alpha}$ be the zeroes of the polynomial.

Since, product of zeroes = $\frac{c}{a}$ for a quadratic polynomial $ax^2 + bx + c$.

So, the reason is true.

$\Rightarrow$ Product of the zeros = $\frac{k}{3}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{k}{3} \Rightarrow k = 3$

Thus, the assertion is false.

Section B

21. $42000 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7$

$= 2^4 \times 3 \times 5^3 \times 7$

$= a^4 \times b \times c^3 \times d$

Then, PIN = dbac = 7325

22.

ADEF is a ||gm $\Rightarrow$ AD||EF and AF||DE

$\Rightarrow$ AB||BF and AC||DE

Using Basic Proportionality theorem,

In $\triangle$ABC, AB||EF $\Rightarrow \frac{CE}{EB} = \frac{CF}{FA}$ ...(1)

Also, AC||DE $\Rightarrow \frac{CE}{EB} = \frac{AD}{DB}$ ...(2)

From (1) and (2)

$\frac{CF}{FA} = \frac{AD}{DB}$

Hence proved.

23. Let the circle touch the sides AB, BC, CD and DA at P, Q, R and S, respectively.

We know that the length of tangents drawn from an external point to a circle are equal.

AP = AS ... (1) {tangents from A}

BP = BQ ... (2) {tangents from B}

CR = CQ ... (3) {tangents from C}

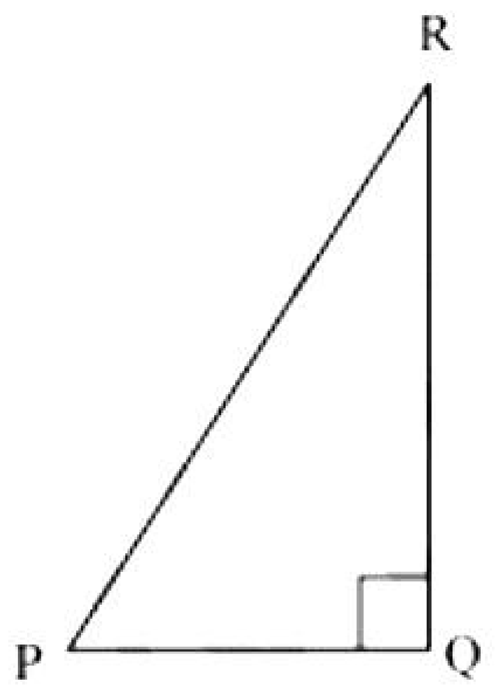
DR = DS ... (4) {tangents from D}
Adding (1), (2), (3) and (4), we get
∴ AP + BP + CR + DR = AS + BQ + CQ + DS
⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
⇒ AB + CD = AD + BC
⇒ AD = (AB + CD) − BC = {(6 + 4) − 7} cm = 3 cm
Hence, AD = 3 cm.

24.

$$\text{L.H.S.} = (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$$
$$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$
$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$
$$= (\sin\theta + \cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)$$
$$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$
$$= \sec\theta + \operatorname{cosec}\theta$$
$$= \text{R.H.S.}$$

OR

Given that PR + QR = 25 cm and PQ = 5 cm
Let PR = x
So, QR = 25 − x



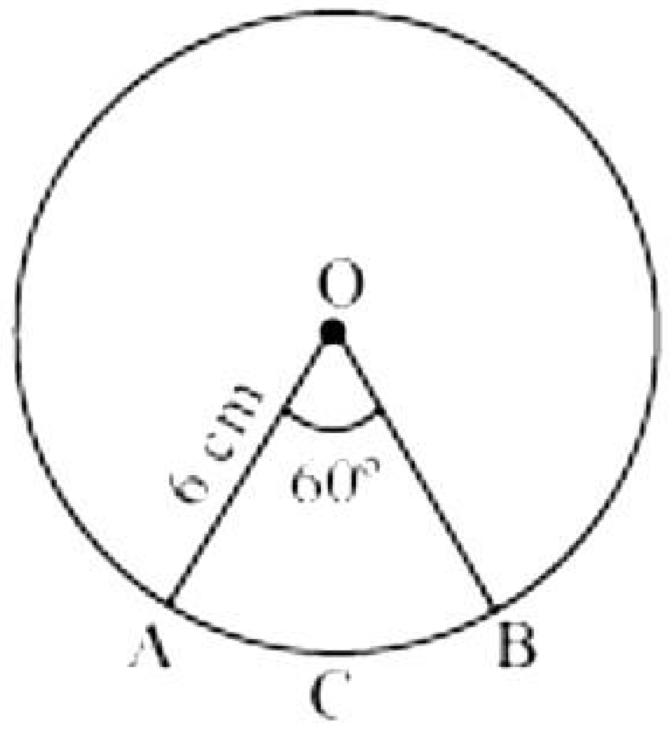
Now applying Pythagoras theorem in ΔPQR,
$PR^2 = PQ^2 + QR^2$
$x^2 = (5)^2 + (25 - x)^2$
$x^2 = 25 + 625 + x^2 - 50x$
$50x = 650 \Rightarrow x = 13$
So, PR = 13 cm
QR = 25 − 13 = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

25.



Let OACB be a sector of circle making 60° angle at centre O of the circle.

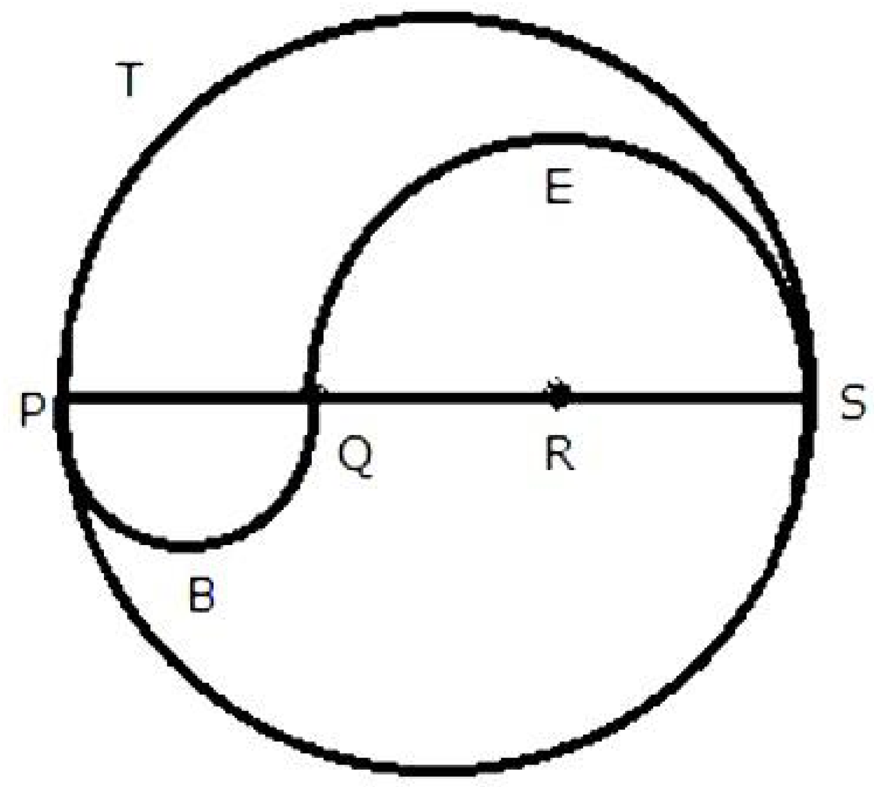
Area of sector of angel $\theta = \frac{\theta}{360°} \times \pi r^2$

So, area of sector OACB = $\frac{60°}{360°} \times \frac{22}{7} \times (6)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7}$ cm^2

So, the area of sector of circle making 60° at the centre of a circle is $\frac{132}{7}$ cm^2.

OR



PS = 12 cm

PQ = QR = RS = 4 cm, QS = 8 cm

Perimeter of the shaded region = arc PTS + arc PBQ + arc QES

$= (\pi \times 6 + \pi \times 2 + \pi \times 4)$ cm

$= 12\pi$ cm

$= 12 \times 3.14$ cm

$= 37.68$ cm

Area of the shaded region = (area of semi-circle PBQ) + (area of semi-circle PTS)

$$= \left[\frac{1}{2}\pi \times (2)^2 + \frac{1}{2} \times \pi \times (6)^2 - \frac{1}{2} \times \pi \times (4)^2\right] cm^2$$

$$= [2\pi + 18\pi - 8\pi] = 12\pi \ cm^2 = (12 \times 3.14) cm^2$$

$$= 37.68 \ cm^2$$

Section C

26. To find minimum number of baskets, we need to first find the maximum and equal number of fruits of same kind to be kept in each basket.

That is, HCF of 50, 90 and 110.

$50 = 2 \times 5 \times 5$

$90 = 2 \times 3 \times 3 \times 5$

$110 = 2 \times 5 \times 11$

Therefore, HCF (50, 90, 110) = $2 \times 5 = 10$

So, minimum number of baskets required $= \frac{50+90+110}{10} = \frac{250}{10} = 25$

27. For an A.P., 3, 8, 13, 18, ...

First term, a = 3

Common difference, $d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this A.P. be 78.

Now, n^{th} term of an A.P. is given by

$a_n = a + (n - 1)d$

$\Rightarrow 78 = 3 + (n - 1)5$

$\Rightarrow 75 = (n - 1)5$

$\Rightarrow n - 1 = 15$

$\Rightarrow n = 16$

Hence, the 16^{th} term of the given A.P. is 78.

28. For an A.P., 11, 8, 5, 2, ...

First term, a = 11

Common difference, $d = a_2 - a_1 = 8 - 11 = -3$

Let −150 be the n^{th} term of this A.P.

Now, n^{th} term of an A.P. is given by

$a_n = a + (n - 1)d$

$\Rightarrow -150 = 11 + (n - 1)(-3)$

$\Rightarrow -161 = -3n + 3$

$\Rightarrow 164 = 3n$

$\Rightarrow n = \frac{164}{3}$

Clearly, n is not an integer.

Therefore, −150 is not a term of the given A.P.

OR

Given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n - 1)d$

$a_{11} = a + (11 - 1)d$

$38 = a + 10d$ ....(1)

Similarly, $a_{16} = a + (16 - 1)d$

$73 = a + 15d$ ....(2)

On subtracting equation (1) from equation (2), we obtain

35 = 5d

⇒ d = 7

From equation (1),

38 = a + 10 × (7)

⇒ a = −32

Therefore, 31st term is given by

$a_{31} = a + (31 - 1)d$

= −32 + 30(7)

= −32 + 210

= 178

Hence, 31st term of the given A.P. is 178.

29. x = cot A + cos A and y = cot A − cos A

Thus, we have

x + y = (cot A + cos A) + (cot A − cos A) = 2cot A

x − y = (cot A + cos A) − (cot A − cos A) = 2cos A

$$\text{L.H.S.} = \left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2$$

$$= \left(\frac{2\cos A}{2\cot A}\right)^2 + \left(\frac{2\cos A}{2}\right)^2$$

$$= \left(\frac{\cos A}{\cot A}\right)^2 + (\cos A)^2$$

$$= \left(\frac{\cos A}{\cos A / \sin A}\right)^2 + (\cos A)^2$$

$$= (\sin A)^2 + (\cos A)^2$$

$$= \sin^2 A + \cos^2 A$$

$$= 1$$

$$= \text{R.H.S.}$$

30. We know that in 1 hour (i.e. 60 minutes) minute hand rotates 360°.

So, in 10 minutes, the minute hand will rotate $= \frac{360^\circ}{60} \times 10 = 60^\circ$

The area swept by minute hand in 10 minutes will be the area of a sector of 60° in a circle of 14 cm radius.

Area of sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Area of sector of 60° $= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$$= \frac{22 \times 14}{3}$$

$$= \frac{308}{3}\ \text{cm}^2$$

So, the area swept by minute hand in 10 minutes is $\frac{308}{3}$ cm^2.

OR

AD = 2 × radius = 2 × 12 = 24 cm

AB = BC = CD = $\frac{24}{3}$ = 8 cm

BD = BC + CD = 8 + 8 = 16 cm

Perimeter of the shaded region

= Length of an arc ATD + Length of an arc ARB + Length of an arc BSD

= (π × 12 + π × 4 + π × 8) cm

= 24π cm

= 24 × 3.14 cm

= 75.36 cm

Area of the shaded region

= Area of semi-circle ARB + Area of semi-circle ATD − Area of semi-circle BSD

$= \left[\frac{1}{2}\pi \times (4)^2 + \frac{1}{2} \times \pi \times (12)^2 - \frac{1}{2} \times \pi \times (8)^2\right] \text{cm}^2$

= (8π + 72π − 32π) cm^2

= 48π cm^2

= 48 × 3.14 cm^2

= 150.72 cm^2

31. 2 black kings, 2 black queens and 2 black jacks are removed from a deck of 52 cards.

Then, remaining number of cards = 52 − 6 = 46

i. As 2 black jacks are removed, only 2 red jacks are left.

∴ Probability of getting a jack card = $\frac{2}{46} = \frac{1}{23}$

ii. When 6 black cards are removed, number of black cards left = 26 − 6 = 20

∴ Probability of getting a black card = $\frac{20}{46} = \frac{10}{23}$

iii. There are 13 heart cards in a deck.

∴ Probability of getting a heart card = $\frac{13}{46}$

Section D

Section D consists of 4 questions of 5 marks each.

32. Let the present age of Rehman be x years.

Three years ago, his age was (x – 3) years.

Five years hence, his age will be (x + 5) years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow 3(2x+2)=(x-3)(x+5)$$

$$\Rightarrow 6x+6=x^2+2x-15$$

$$\Rightarrow x^2-4x-21=0$$

$$\Rightarrow x^2-7x+3x-21=0$$

$$\Rightarrow x(x-7)+3(x-7)=0$$

$$\Rightarrow (x-7)(x+3)=0$$

$$\Rightarrow x=7,-3$$

Since age cannot be negative, x = 7

Therefore, Rehman's present age is 7 years.

OR

Let the larger and smaller numbers be x and y respectively.
According to the given question,

$x^2 - y^2 = 180$ and $y^2 = 8x$

$\Rightarrow x^2 - 8x = 180$

$\Rightarrow x^2 - 8x - 180 = 0$

$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$\Rightarrow x(x-18) + 10(x-18) = 0$

$\Rightarrow (x-18)(x+10) = 0$

$\Rightarrow x = 18, -10$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.

Therefore, the larger number will be 18 only.

$\Rightarrow x = 18$

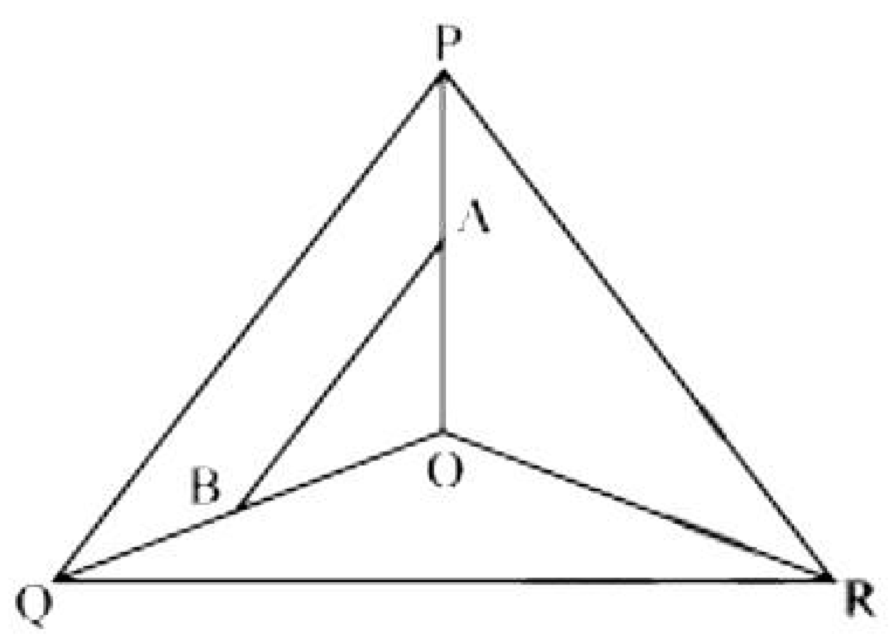
$\therefore\ y^2 = 8x = 8 \times 18 = 144$

$\Rightarrow\ y = \pm\sqrt{144} = \pm 12$

$\therefore$ Smaller number $= \pm 12$

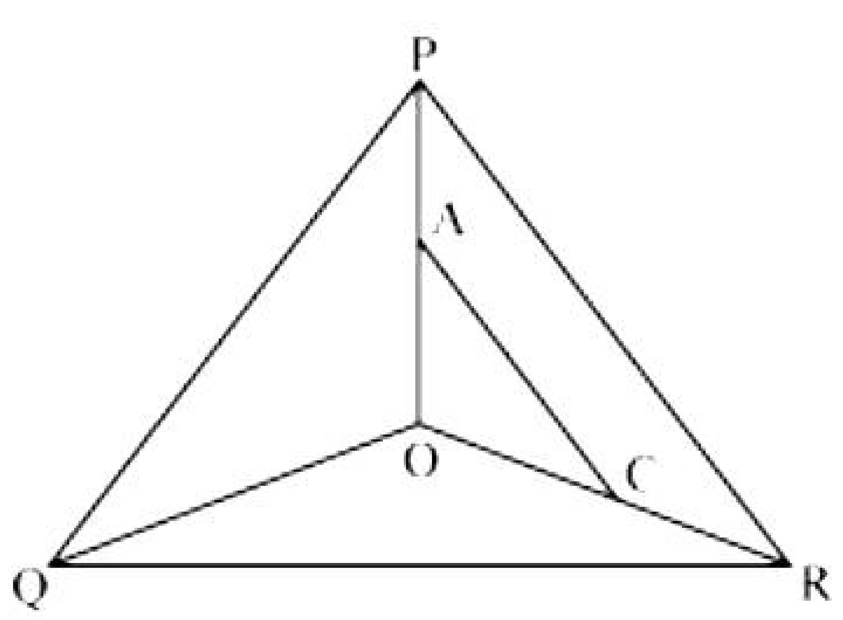
Therefore, the numbers are 18 and 12 or 18 and −12.

33.



In ΔPOQ, AB || PQ,

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ (i) [By basic proportionality theorem]



In ΔPOR, AC || PR,

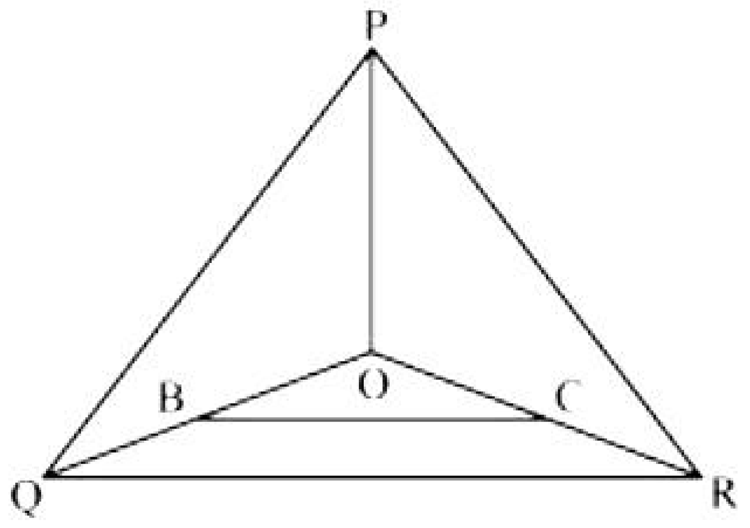
$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad (ii) \quad \text{[By basic proportionality theorem]}$$

From (i) and (ii)

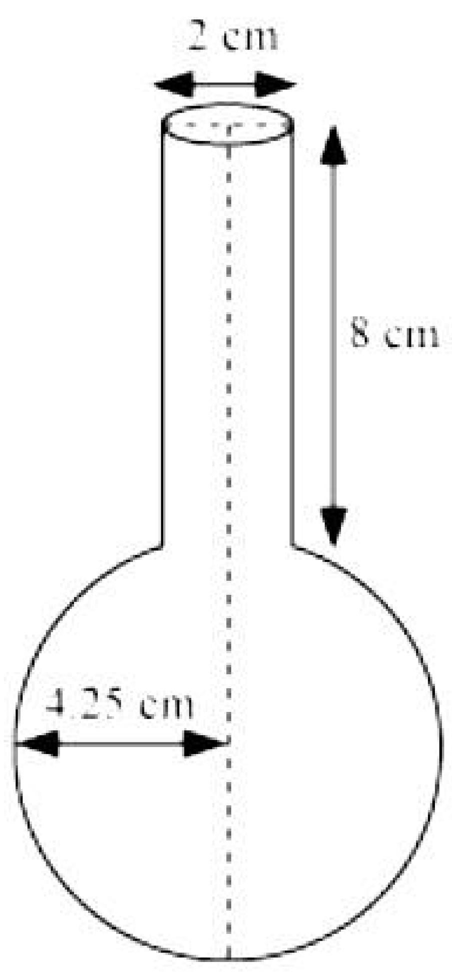
$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Therefore BC || QR

(By converse of basic proportionality theorem)



34.



Radius (r_1) of spherical part = 8.5/2

Height (h) of cylindrical part = 8 cm

Radius (r_2) of cylindrical part = $\frac{2}{2} = 1$ cm

Volume of vessel = volume of sphere + volume of cylinder

$$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h$$

$$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi(1)^2(8)$$

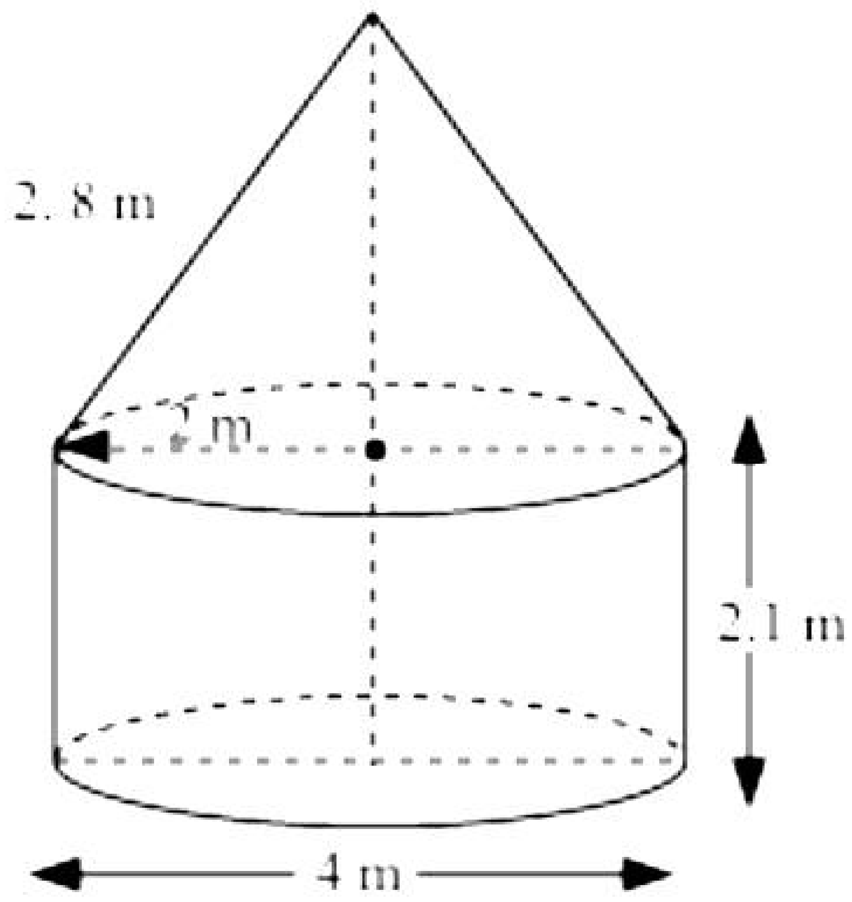
$$= \frac{4}{3} \times 3.14 \times \left(\frac{8.5}{2}\right)^3 + 3.14 \times 8$$

$$= 321.39 + 25.12$$

$$= 346.51 \text{ cm}^3$$

Hence, she is wrong.

OR



Given that
Height (h) of the cylindrical part = 2.1 m
Diameter of the cylindrical part = 4 m
So, radius (r) of the cylindrical part = 2 m
Slant height (l) of conical part = 2.8 m
Area of canvas used = CSA of conical part + CSA of cylindrical part

$$= \pi rl + 2\pi rh$$
$$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$$
$$= 2\pi[2.8 + 4.2]$$
$$= 2 \times \frac{22}{7} \times 7$$
$$= 44 \text{ m}^2$$

Cost of 1 m^2 canvas = Rs. 500
Cost of 44 m^2 canvas = Rs. (44 × 500) = Rs. 22000
So, it will cost Rs. 22000 for making such tent.

35. Taking 16 as assumed mean (a) we may calculate d_i and f_id_i as following–

Number of days	Number of students f_i	x_i	$d_i = x_i - 17$	f_id_i
0 – 6	11	3	−14	−154
6 – 10	10	8	−9	−90
10 – 14	7	12	−5	−35
14 – 20	4	17	0	0
20 – 28	4	24	7	28
28 – 38	3	33	16	48
38 – 40	1	39	22	22
Total	40			−181

Now, $\sum f_i = 40$ and $\sum f_i d_i = -181$

mean $\bar{x} = a + \left(\frac{\sum f_i d_i}{\sum f_i}\right)$

$$= 17 + \left(\frac{-181}{40}\right)$$

$$= 17 - 4.525$$

$$= 12.475$$

$$\approx 12.48$$

So, mean number of days is 12.48 days, for which a student was absent.

Section E

36.

i. From the graph, the coordinates of points A and B are (1, 1) and (4, 1) respectively.

$\therefore AB = \sqrt{(1-4)^2 + (1-1)^2} = 3$ km

ii. From the graph, the coordinates of points B and C are (4, 1) and (4, 5) respectively.

$\therefore BC = \sqrt{(4-4)^2 + (5-1)^2} = 4$ km

iii. From the graph, the coordinates of points A and C are (1, 1) and (4, 5) respectively.

$\therefore AC = \sqrt{(4-1)^2 + (5-1)^2} = 5$ km

OR

From the graph, the coordinates of points B and D are (4, 1) and (7, 5) respectively.

$\therefore BD = \sqrt{(7-4)^2 + (5-1)^2} = 5$ km

37.

i. The given situation can form the A.P.

Here,

Rate for first km = Rs. 15 = a = first term of A.P.

Rate for additional km = Rs. 8 = d = common difference

Travel fare for 4 km = a_4 = fourth term

$a_n = a + (n-1)d$

$a_4 = 15 + (4-1)8 = 39$

Therefore, the taxi fare for 4 km will be Rs. 39.

ii. Rate for first km = Rs. 15 = a = first term of A.P.

Rate for additional km = Rs. 8 = d = common difference

Travel fare = 63 = a_n

$a_n = a + (n-1)d$

$\Rightarrow 63 = 15 + (n-1)8$

$\Rightarrow 48 = (n-1)8$

$\Rightarrow n - 1 = 6$

$\Rightarrow n = 7$

Therefore, for taxi fare to be Rs. 63, the total distance travelled will be 7 km.

OR

Rate for first km = Rs. 15 = a = first term of A.P.

Rate for additional km = Rs. 8 = d = common difference = d

$a_n = a + (n-1)d$

$\Rightarrow 103 = 15 + (n-1)8$

$\Rightarrow 88 = (n-1)8$

$\Rightarrow n - 1 = 11$

$\Rightarrow n = 12$

Therefore, the distance from place A to B is 12 km.

iii. Rate for first km = Rs. 15 = a = first term of A.P.

Rate for additional km = Rs. 8 = d = common difference = d

Difference in the fare for 5 km and 3 km = $a_5 - a_3$

$a_5 - a_3 = [15 + (4)8] - [15 + (2)8]$

$= 15 + 32 - 15 - 16$

$= 16$

38.

i.

In ΔABH,

$$\tan 45^\circ = \frac{AB}{BH}$$

$$\Rightarrow 1 = \frac{10}{BH}$$

$$\Rightarrow BH = 10 \text{ m}$$

Therefore, the distance between Harsh and building AB is 10 m.

ii.

In ΔCDH,

$$\tan 60^\circ = \frac{CD}{DH}$$

$$\Rightarrow \sqrt{3} = \frac{15}{DH}$$

$$\Rightarrow DH = \frac{3 \times 5}{\sqrt{3}}$$

$$\Rightarrow DH = 5\sqrt{3} \text{ m}$$

Therefore, the distance between Harsh and building CD is $5\sqrt{3}$ m.

iii.

In ΔABH,

$$\sin 45^\circ = \frac{AB}{AH}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{AH}$$

$$\Rightarrow AH = 10\sqrt{2} \text{ m}$$

Therefore, the length of a rope joining the points A and H is $10\sqrt{2}$ m.

OR

In ΔCDH,

$$\sin 60^\circ = \frac{CD}{HC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{15}{HC}$$

$$\Rightarrow HC = \frac{15 \times 2}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

Therefore, the length of a rope joining the points C and H is $10\sqrt{3}$ m.